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DISTANCE-BASED ASSESSMENT OF DEFECT DETECTION PROBABILITY IN CRITICAL ZONES OF AN ELECTRIC ROLLING STOCK BOGIE FRAME

Adilov Nodir Botir ugli
Toshtemirov Humoyun Shokir ugli
Tashkent State Transport University (Tashkent, Uzbekistan)

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Abstract. The paper addresses the statistical relationship between travelled distance and the first detection of a structural defect in a predefined critical zone of an electric rolling stock bogie frame. In periodic non-destructive testing, the exact distance at which a defect becomes detectable is generally unknown; the event is therefore represented by the interval between the last negative and the first positive inspection. A two-parameter Weibull model is formulated for interval-, right- and left-censored observations, with parameters to be estimated by maximum likelihood when verified inspection records become available. Turnbull's nonparametric estimate is proposed as a reference for checking the parametric assumption. The Weibull survival function is then used to derive the conditional defect risk after a current inspection and an analytical expression for the next inspection distance under a predefined allowable risk. The study is methodological and does not substitute synthetic values for missing fleet data.

Keywords: electric rolling stock, bogie frame, critical zone, fatigue crack, travelled distance, interval censoring, Weibull distribution, non-destructive testing, reliability, inspection interval.

Introduction

The bogie frame is one of the most critical load-bearing components of electric rolling stock, simultaneously withstanding vertical loads transmitted from the carbody, traction and braking forces, wheel–rail interaction forces, and suspension system reactions. The repeated and variable nature of these loads can lead to the accumulation of fatigue damage, particularly in zones containing welded joints and geometric stress concentrators. Therefore, assessment of the technical condition of the frame should rely not only on a one-time strength calculation but also on the history of operational loading and periodic diagnostic data [1, 2].

Standard load cases, measured load spectra, and simulated operating conditions are used to assess the fatigue life of bogie frames. Xiu et al. compared these approaches and demonstrated the advantages of using actual load spectra, while also noting that obtaining such data requires extensive measurements and data processing [2]. Qu et al. showed that changes in dynamic behaviour during long-term operation can affect stress distribution in the frame and may be associated with fatigue cracking [3]. These studies indicate that travelled distance should be regarded not as the sole physical cause of cracking, but as an indicator of operating duration that is consistently recorded in service systems.

It is also important to distinguish between computational and diagnostic tasks. Finite element analysis identifies zones of high stress or high stress gradient, but does not directly determine when such a zone enters a defective state under actual operating conditions. Seo et al. analysed crack

growth under dynamic stresses using FEM and fracture mechanics [4], while Yan et al. demonstrated the feasibility of monitoring cracks in welded zones using ultrasonic guided waves [5]. Thus, a separate statistical relationship is needed between a computationally identified critical zone and actual NDT results.

Periodic NDT presents another practical challenge: the exact travelled distance at which a crack or other structural defect reaches the detection threshold is generally unknown. What is known is that no defect was recorded at the previous inspection, whereas the defect was detected at the subsequent inspection. In reliability analysis, such an observation is considered interval-censored data. Turnbull proposed a nonparametric distribution estimator for censored data [6], while Finkelstein developed a proportional hazards model for interval-censored observations [7].

The aim of this study is to formulate a mathematical model relating travelled distance to the first detection of a defect in a predefined critical zone of an electric rolling stock bogie frame. The main emphasis is on constructing a likelihood function consistent with the structure of actual service records, checking the model, and determining the next inspection interval according to a risk criterion after the parameters have been estimated from real observations.

Problem Formulation and Observation Data

The "frame–critical zone" pair is taken as the unit of analysis. The critical zone must not be selected retrospectively on the basis of the statistical result; rather, it should be predefined from FEM calculations, maps of previously observed defects, weld geometry, or other technical justification. This requirement reduces selection bias during model development.

Let i = 1, …, n denote the objects and k = 1, …, K denote the critical zones. The random variable T_{ik} represents the distance travelled by object i until a structural defect in zone k reaches the detection threshold of NDT. This definition is not equivalent to the microscopic moment of physical crack initiation; it is linked to an event observable in service practice, namely the first recording of the defect.

Under periodic inspection, the event can be observed in three forms. First, if a defect was absent at the preceding inspection but present at the subsequent one, the event belongs to the interval between those two travelled-distance values. Second, if no defect is detected up to the final observation point, the observation is right-censored. Third, if a defect is recorded at the first available inspection and its prior status is unknown, the observation is left-censored.

Table 1. Minimum Data Required for Model Identification

Field	Symbol	Description
Frame/object identifier	ID	To link repeated observations
Critical zone code	k	Predefined diagnostic zone
Travelled distance at inspection	L, km	Distance axis of the survival model
NDT result	D	0 – not detected; 1 – detected
NDT method	M	UT, MT, or another method
Defect size	a	Recorded if available
Inspection date	date	To reconstruct operating

Field	Symbol	Description
		history
Repair/replacement	R	Indicates whether the risk history is reset
Operational factors	X	If available: route, load, speed, etc.

Negative inspection results must also be retained in the actual dataset. Collecting only cases in which defects were found violates the fundamental logic of the survival model, because objects observed up to a known distance without defect detection also provide information for parameter estimation.

Mathematical Model

A two-parameter Weibull distribution is selected as the initial parametric model. This choice is motivated by its widespread use in reliability analysis and by the fact that the shape of the hazard function is governed by a single parameter [8–11]. Here, $L \geq 0$ is the travelled distance, $\eta > 0$ is the scale parameter, and $\beta > 0$ is the shape parameter.

$$F(L) = P(T \leq L) = 1 - \exp[-(L/\eta)^{\beta}] \quad (1)$$

F(L) represents the cumulative probability that a defect in the critical zone reaches an NDT-detectable state by distance L. The corresponding survival function is:

$$S(L) = P(T > L) = \exp[-(L/\eta)^{\beta}] \quad (2)$$

The hazard function, which expresses the event intensity as distance increases conditional on the current defect-free state, is given by:

$$h(L) = (\beta/\eta)(L/\eta)^{\beta-1} \quad (3)$$

The physical and statistical interpretation of parameter β should be made after the results have been obtained. If $\beta > 1$, the hazard increases with travelled distance; if $\beta = 1$, the hazard is constant; and if $\beta < 1$, the hazard decreases. It would be inappropriate to predetermine which case applies to the bogie frame; this must be established from actual data.

Likelihood Function for Censored Observations

If the last negative inspection is recorded at distance L_i^- and the first positive inspection at distance L_i^+, the probability that the event falls within this interval is expressed as:

$$P(L_i^- < T_i \leq L_i^+) = S(L_i^-) - S(L_i^+) \quad (4)$$

For a right-censored observation, no defect has been found up to the last known distance:

$$P(T_i > L_i) = S(L_i) \quad (5)$$

For a left-censored observation:

$$P(T_i \leq L_i) = F(L_i) = 1 - S(L_i) \quad (6)$$

Let I, R, and C denote the sets of interval-censored, right-censored, and left-censored observations, respectively. The overall likelihood function is:

$$\mathscr{L}(\beta,\eta) = \prod_{i\in I}[S(L_i^-)-S(L_i^+)] \cdot \prod_{i\in R}S(L_i) \cdot \prod_{i\in C}[1-S(L_i)] \quad (7)$$

The parameters are estimated using the maximum likelihood criterion:

$$(\hat{\beta}, \hat{\eta}) = \arg\max \ln \mathscr{L}(\beta,\eta), \quad \beta>0, \eta>0 \quad (8)$$

In practical computation, using the log-likelihood improves numerical stability. Parameter uncertainty can be assessed by profile likelihood or bootstrap methods. The Weibull assumption should not be imposed on the data without verification: graphical comparison with Turnbull's nonparametric estimate [6] for interval-censored observations is a straightforward and justified way to assess the assumed model shape.

Risk-Based Determination of the Inspection Interval

After the Weibull model has been identified using actual data, the main service question can be stated more precisely than in terms of cumulative probability: if the critical zone is assessed as defect-free at the current inspection distance L_0, what is the probability that the defect will reach a detectable state over the next interval ΔL?

$$P(L_0 < T \le L_0+\Delta L \mid T > L_0) = 1 - S(L_0+\Delta L)/S(L_0) \quad (9)$$

Substituting the Weibull survival function gives:

$$pcond(L_0,\Delta L) = 1 - \exp\{-[((L_0+\Delta L)^\beta - L_0^\beta)/\eta^\beta]\} \quad (10)$$

Let p^* be an allowable conditional risk justified by the operator or a regulatory document. Under the condition $pcond \le p^*$, the theoretical maximum distance until the next inspection is:

$$\Delta L_{max} = [L_0^\beta - \eta^\beta \ln(1-p^*)]^{1/\beta} - L_0 \quad (11)$$

Expression (11) does not automatically declare the inspection interval "safe". The value of p^* must be selected with reference to the consequences of a defect, the detection capability of the NDT method used, operating conditions, and the operator's safety policy. Furthermore, the formula acquires practical significance only after β and η have been estimated robustly from actual data and the model diagnostics have produced satisfactory results.

The practical sequence of the study is as follows: the critical zone is predefined; the zone is monitored by periodic NDT; the L^- and L^+ boundaries are constructed from inspections; the censoring type is identified; the Weibull parameters are estimated; model fit is assessed; and only then is the inspection interval calculated according to the risk criterion.

Model Validation and Conditions for Practical Application

A single measure is insufficient to establish model reliability. First, the Weibull curve is compared graphically with Turnbull's nonparametric survival estimate. If the sample size permits, the Weibull model is compared with lognormal or log-logistic distributions using AIC/BIC and likelihood criteria. Confidence intervals for β and η must be reported separately.

Travelled distance is not a complete physical equivalent of fatigue damage. The same distance may lead to different levels of accumulated damage under different routes, loads, speeds, and traction–braking regimes. If such operational indicators are recorded with sufficient quality, covariates can be introduced into the model at a subsequent stage. However, constructing a multivariable model when data are insufficient leads to parameter instability and overfitting.

An NDT result is not an absolute observation either. The probability of detection depends on defect size, orientation, surface condition, the operator, and the instrument. Accordingly, the present model describes not the first physical initiation of a crack but its first detection by the inspection system actually employed. If the probability of detection (POD) is known, it can be included in sensitivity analysis; otherwise, this operational meaning of the result must remain explicit.

Post-repair observations must also be considered separately. When a crack is restored by welding, the zone is reworked, or the frame is replaced, it may be incorrect to automatically combine the previous risk history with the new period. If recurrent events become more frequent, it will be necessary to move from a first-event Weibull model to recurrent-event or frailty models.

Discussion

The main purpose of the proposed approach is not simply to apply the Weibull distribution to a bogie frame. Its scientific and methodological value lies in linking physical and service data in a single sequence: the critical zone is selected in advance on the basis of calculations or design considerations, monitored through periodic NDT, and documented as interval observations indexed by travelled distance; only then is the probabilistic model developed. This sequence gives the statistical model a physical interpretation.

Treating the result of periodic NDT as evidence that “the defect appeared at exactly this distance” may be a methodological error. The first positive inspection identifies not the exact event location on the distance axis, but only that the event occurred after the preceding negative inspection. Interval-censored likelihood does not conceal this uncertainty and enables appropriate use of the available data.

Expression (11) yields an interesting result from the service perspective: if $\beta > 1$, the interval until the next inspection may decrease as the current travelled distance increases, for the same allowable risk. This provides a mathematical basis for moving from inspections at strictly uniform intervals to differentiated, risk-based inspection. However, this conclusion can be applied only if $\beta > 1$ is confirmed by actual data.

Actual operator NDT logs and travelled-distance histories are not available in this study. Consequently, reporting numerical results for β, η, defect percentage, or a “safe distance” would be scientifically unjustified. The present result is limited to formulating the method and computational sequence; at the next stage, the model must be identified and validated against actual diagnostic databases for critical zones.

Conclusions

Under periodic NDT, the first detection of a defect in a critical zone of the bogie frame is often represented not by an exact distance point but by an interval between two inspections.

Therefore, an interval-censored survival model is consistent with the structure of the actual observation data. A two-parameter Weibull model and a likelihood function combining interval-, right-, and left-censored observations were formulated to assess defect detection probability as a function of travelled distance. The values of β and η must be determined solely from actual observation data. An analytical expression was obtained for conditional risk after a current defect-free inspection, and a formula was presented for calculating the next inspection distance according to a predefined and justified risk level p*. This formula can be used as a practical decision-making tool only after the model has been validated. Travelled distance is not the sole physical measure of operational damage. The next practical stage consists of collecting standardized NDT results, inspection distances, and necessary operating factors in a unified database and validating the model using those data.

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